PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Dust Discs around Low-Mass Main-Sequence Stars

R. D. Wolstencroft and Helen J. Walker

Phil. Trans. R. Soc. Lond. A 1988 **325**, 423-437
doi: 10.1098/rsta.1988.0058

Phil. Trans R. Soc. Lond. A **325**, 423–437 (1988)
Printed in Great Britain

Dust discs around low-mass main-sequence stars

By R. D. Wolstencroft[1] and Helen J. Walker[2]

[1] *Royal Observatory, Edinburgh EH9 3HJ, U.K.*

[2] *SETI Institute, NASA Ames Research Center, Moffett Field, California* 94035, *U.S.A.*

Current understanding of the formation of circumstellar discs as a natural accompaniment to the process of low-mass star formation is briefly reviewed. Models of the thermal emission from the dust discs around the prototype stars α Lyr, α PsA, β Pic and ε Eri are discussed, which indicate that the central regions of three of these discs are almost devoid of dust within radii ranging between 17 and 26 AU, with the temperature of the hottest dust lying between about 115 and 210 K. One possible explanation of the dust-free zones is the presence of a planet at the inner boundary of each cloud that sweeps up grains crossing its orbit. The discs have outer radii that range between about 250 and 800 AU and have dust masses that are unlikely to exceed about 300 Earth masses. Assuming a gas:dust ratio of 100:1 for the pre-main-sequence disc this corresponds to a mass of *ca.* 0.1 $M_{\odot}$ comparable to that of the pre-main-sequence star HL Tau. The colour, diameter and thickness of the optical image of β Pic, obtained by coronagraphic techniques, have provided further information on the size, radial distribution of number density and orbital inclination of the grains. The difference in surface brightness on the two sides of the disc is puzzling, but might be explained if the grains are elongated and aligned by the combined effects of a stellar wind and a magnetic field of spiral configuration. Finally, we discuss the orbital evolution and lifetimes of particles in these discs, which are governed primarily by radiation pressure, Poynting–Robertson drag and grain–grain collisions. Although replenishment of these discs may be occurring, for example by grains ejected from comets, discs of initial radius *ca.* 1000 AU can survive Poynting–Robertson depletion over the stellar age and there is no *prima facie* evidence as yet in favour of a balance between sources and sinks of dust.

1. Introduction

There is considerable evidence to support the view that circumstellar discs are a common and perhaps inevitable accompaniment of the low-mass star formation process. Because the Solar System almost certainly evolved from such a circumstellar disc, the formation and evolution of such discs has become a subject of intense interest. The detection by IRAS (infrared astronomical satellite) of far infrared excesses around the main sequence stars α Lyr, β Pic, α PsA and ε Eri (Aumann *et al.* 1984; Gillett 1986; Aumann 1985) and their interpretation in terms of circumstellar discs of dust has stimulated much activity in this field and led to speculation that planetary systems may be forming or have formed around these and similar stars. In this paper we shall review this topic. It is worth noting at the outset that systems in which matter in the disc has formed into planets may be the least likely to be detected in searches for an infrared excess associated with discs. The unique example of this is the zodiacal dust cloud around the Sun which Good (1987) has shown could not be detected at the distance of Vega (8 pc): yet HD207129, another solar type star, is detected and is a good disc candidate (Aumann 1985) at a distance of 14 pc. If the formation of discs as an accompaniment to low-mass star formation is the rule rather than the exception then the best candidate stars for being the site of planetary systems may be those nearby stars in which dust discs are *not* detected. An exception to this 'rule' may prove to be the case of ε Eri, one of the stars known to have a circumstellar disc: high-precision radial velocity measurements of this star taken over an interval of about 6 years suggest the presence of an unseen companion with a mass less than about 10 Jupiter masses, $P \geqslant 6$ years and semimajor axis greater than about 3 AU (Campbell *et al.* 1987).

In §2 of this paper we discuss what is known about the pre-main-sequence evolution of circumstellar discs. In §3 we briefly describe searches for discs associated with nearby main-sequence stars and discuss observations and models for the four well-known prototypes referred to above. The subject of the orbital evolution and replenishment of grains is examined in §4.

2. Young discs

Before discussing the properties of discs around low-mass stars on the main sequence it is useful to outline briefly current understanding of the main stages that lead to the formation of low-mass stars and their surrounding discs. We restrict ourselves here to the scenario of star formation without discussing the extensive literature on the formation and evolution of planets and planetary systems *per se*, which is well covered in the excellent volume *Protostars and planets II* edited by Black & Matthews (1985). The process of star formation as described by Shu & Adams (1987) and Cassen *et al.* (1985) takes place in four phases. In the first phase, a dense core forms in a molecular cloud whose properties, based on observations of NH_3 lines at 1.3 cm by Myers & Benson (1983), have the following typical values: diameter 0.03–0.3 pc, $n \approx 10^4$–10^5 cm^{-3}, $T \approx 10$–15 K, mass *ca.* 4 $M_\odot$ and velocity width *ca.* 0.3 km s^{-1}. The formation of these cores from less dense clumps probably requires the action of ambipolar diffusion to allow magnetic flux, which provides some support against gravity, to leak out of the developing core. At the start of the second phase these cores begin to undergo gravitational collapse. In Shu & Adams's scenario freefall collapse begins at the centre and the region of this collapse moves steadily outwards at the speed of sound. Initially, infalling matter with low angular momentum accretes directly onto the growing protostar, but, as the collapse proceeds, matter of increasingly higher angular momentum, which is generally farther out in the core, will fall in on orbits which pass progressively farther from the protostar: the result is the formation of a disc which grows in radius and mass with time. The rotating protostar and disc continue to gain mass until deuterium-burning begins: this occurs when the protostellar mass is *ca.* 0.3 $M_\odot$. This initiates convection which spreads outwards to encompass almost the entire star by the time the stellar mass is *ca.* 0.5 $M_\odot$. The combination of convection and differential rotation in the star leads to dynamo action (Parker 1979) and to intense magnetic activity at the stellar surface. By a mechanism not yet fully understood a stellar-wind results. Because infall is still taking place the stellar wind cannot escape initially, but eventually this happens and the infalling cloud is punctured and a narrow bipolar outflow results. In this third phase infall and outflow are occurring simultaneously: the very cold source IRAS 16293–2422 was recently shown to be a very young star at this evolutionary stage (Walker *et al.* 1986). Matter continues to fall onto the disc and at the same time the opening angle of the wind steadily widens and eventually sweeps out in all directions halting any further infall. At this point, the start of the fourth phase, the star should be visible from almost any direction as a 'naked' newly born T Tauri star, evolving along its convective pre-main sequence track.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Cassen *et al.* (1985) discuss the evolution of these discs, which is determined principally by the mass, M_d, angular momentum, J_d, and kinematic viscosity, ν, of the disc. The disc will have a size consistent with the balance of centrifugal and gravitational forces unless it spreads to even larger sizes by viscous diffusion. The radius of such a disc is $R_{cf} = J_d^2/k^2GM_d^3$, where k is a constant of order unity. Such discs are likely to be massive with $M_d \sim M_*$ (Cassen & Summers 1983) and subject to gravitational instability. A possible consequence of this instability, if spiral density waves do not regulate mass and momentum transport in the disc, would be the formation of a binary or multiple star system. If instead viscous forces dominate in the disc, as measured by the viscous radius $R_v = (\nu t)^{\frac{1}{2}}$ (t is the accretion time), then the outer parts of the disc expand as angular momentum is transported outwards while matter in the inner regions of the disc spirals inwards and falls on to the star. Thus the star gains mass not only by direct infall but also by accretion from the disc and as a result the disc mass is small, namely $M_d \sim (R_{cf}/R_v)^{\frac{1}{2}}\ M_* \ll M_*$. Such discs are more likely to be the site of planetary-system formation than are the centrifugal discs. Finally, stars for which the protostellar radius is greater than either R_{cf} or R_v will form no discs.

Little hard evidence is available on the properties of pre-main-sequence discs. One notable exception is the nearby pre-main-sequence star, HL Tau, whose disc properties have been measured in the $J = 1$–0 transition of ^{13}CO by the Millimeter Wave Interferometer at the Owens Valley Radio Observatory (Beckwith & Sargent 1987). Maps with a resolution of $6 \times 10''$ obtained at several velocities, one of which is the stellar velocity, reveal that (*a*) the emission integrated over velocity is centred on the star and is elongated in the form of a disc of diameter 30″ and position angle 146° E of N; and (*b*) the ^{13}CO maxima at velocities of 0.52 km s^{-1} either side of the stellar velocity peak lie on opposite sides of the star at a radial distance along the disc axis of about 10″. After allowance for the effects of finite velocity and angular resolution they estimate that the total mass of gas in the disc (assuming the standard conversion from CO to H_2 column density) is between 0.01 and 0.5 $M_\odot$ with a most probable value of 0.1 $M_\odot$, and that the disc radius is about 2000 AU. The four velocity measurements are shown to be consistent with gas in keplerian orbits ($v \propto r^{-0.5}$) about a star of mass = 1 $M_\odot$. Beckwith & Sargent (1987) also measured the 2.7 mm continuum emission from HL Tau with the interferometer: the measured emission, which is very probably thermal emission from grains in the surrounding dust cloud, is unresolved but can be used to determine the total mass (not just the mass of dust) surrounding HL Tau using the method of Hildebrand (1983). They deduce a total mass in the range 0.08–0.9 $M_\odot$, which is consistent with the mass estimate based on ^{13}CO. The presence of a dust disc surrounding HL Tau was suggested by Cohen (1983) who argued (see also Rydgren & Cohen 1985) from the deep 10 μm silicate absorption that the disc is being viewed edge-on. An optical jet 20″ long and at a position angle 36°±3° E of N was detected by Mundt & Fried (1983): the jet is not orthogonal to the disc detected in CO but inclined at an angle of 70°±3°. This is reminiscent of the geometry of the disc surrounding the pre-main-sequence star R Cr A where the angle between jet and disc is 59°±7° (Ward-Thompson *et al.* 1985); another similarity between the jets of these two stars is that the line of the jet does not pass through the star but is displaced a few seconds of arc from the star. For R Cr A, Ward-Thompson *et al.* proposed a model to explain these features in which the extended gas disc is inclined by about 30° to the inner circumstellar dust disc, which is responsible for collimating the optical jet. Perhaps this model applies also to HL Tau.

Many and perhaps most of the dust grains found in molecular clouds, which ultimately populate the discs around pre-main-sequence stars, were formed originally in the envelopes of low and intermediate mass stars after they had left the main sequence. These red-giant stars lose mass at a prodigious rate (up to $10^{-4}\ M_\odot\ a^{-1}$) during their ascent of the asymptotic giant branch (AGB) and dust grains form in these outflows. The various types of AGB star such as oxygen-rich and carbon-rich Mira variables, OH/IR stars and S stars, may produce grains of significantly different composition, and the spatial distribution of these stars in the Galaxy will play a role in determining the heterogeneity and mean composition of grains in a given molecular cloud and eventually in individual stellar discs.

3. Main sequence discs: observations and models

A dust grain in the vicinity of a star acquires an equilibrium temperature that depends on the temperature of the star, the emissivity of the grain and its distance to the star. The thermal radiation from a circumstellar ensemble of grains gives rise to an infrared excess, which adds to the radiation from the stellar photosphere, and the integrated spectrum of the star plus cloud can in principle provide information on the size, mass and temperature of the cloud. Large infrared excesses are seen in a variety of pre- and post-main sequence stars but until the discovery of the infrared excess around Vega (Aumann *et al.* 1984) circumstellar dust around main sequence stars was unknown, apart from the obvious exception of the Sun.

In this section we describe what is known about the properties of the dust discs that ultimately arrive at the main sequence following the complex process of star formation and pre-main sequence evolution outlined in §2. It is generally accepted that the dust grains in these discs are the survivors of this process and that the winds from these low-mass dwarf stars are too small to add to the dust already present. For Vega an upper limit to the mass loss rate of $\dot{M} < 3.4 \times 10^{-10}\ M_\odot\ a^{-1}$ derived by Hollis *et al.* (1985) is much too small for grain formation to be likely. Thus it is reasonable to assume that the dust grains in these discs are in orbit about the star.

A systematic search for main-sequence stars with Vega-like discs was done by Aumann (1985). He examined the IRAS point source catalogue looking for sources having a good positional association with known dwarf or subgiant stars within 25 pc in the catalogues of Gliese (1969) and Woolley *et al.* (1970). Using the infrared excess of Vega (which peaks at 60μm) as a guide, he found 36 sources with measured flux at 12, 25 and 60 μm and a significant excess at 60 μm. Of these, 23 have a [12]–[60] excess of less than 0.35 and 16 have [12]–[60] > 1.0 (the square bracket indicates the magnitude at that wavelength). The surface density of IRAS sources close to the galactic plane is very high and the chance of accidental positional coincidence between an IRAS source and an unrelated star is significant; therefore Aumann chose to confine his Vega-like objects to those 12 stars with galactic latitude greater than 10° and [12]–[60] > 1.0. Apart from Vega, this group of 12 contains β Pic, α PsA and ε Eri. These three stars together with Vega are the best studied examples of this class of object (we label them the prototypes) and below we discuss the observations and models for these stars in some detail. Later in the paper we briefly examine the properties of the eight remaining objects of Aumann's list as well as other objects similar to the prototypes which have been identified in the IRAS database by Sadakane & Nishida (1986) and Walker & Wolstencroft (1988) using different selection criteria. Before turning to the group of four stars we discuss the interpretation of the optical imaging of β Pictoris.

Coronagraphic imaging of β pictoris

An optical image of the disc around β Pic was first obtained by Smith & Terrile (1984). They used a circular mask of 7″ diameter to occult much of the direct starlight and a Lyot stop to strongly reduce the scattered light associated with the telescope optics and secondary mirror support structure. Although this coronagraphic arrangement is effective, the disc must still be viewed against a halo of atmospherically scattered light whose intensity can vary with time as the number of forward-scattering aerosols in the line of sight changes. In the case of Smith & Terrile's observations at 8900 Å the stellar disc was about a factor of four fainter than the atmospheric halo. Division of the β Pic image by the nearby stellar point source α Pic (which has no infrared excess) revealed a disc which is clearly seen edge on at a position angle of 29° E of N and is visible out to at least 25″ or 400 AU from the star. This experiment has since been repeated by Gradie *et al.* (1987) and Paresce & Burrows (1987): the former group find that the disc at 0.9 μm wavelength extends out to at least 650 AU at 29° E of N and to at least 500 AU at 29° W of S. (Note that the images of β Pic obtained by Gradie *et al.* and by Paresce & Burrows, as well as an image obtained by Terrile & Smith in 1985, are shown in *Sky and Telescope* **73**, 274 (1987).)

The asymmetry of the two sides of the disc, which was first noted by Smith & Terrile, is puzzling. If the grains are spherical and their spatial distribution depends only on distance to the star it is difficult to see how it could arise. One explanation recently suggested by Whitmire *et al.* (1987) is that β Pic has a brown dwarf companion in an eccentric orbit: this orbit would perturb the grain orbits and could lead to a density distribution which depends on longitude and, given a favourable viewing geometry, might produce a significant asymmetry in the column density on the two sides of the disc. An alternative explanation involving grain alignment of elongated grains could in principle provide the asymmetry. An example of the alignment geometry required is that postulated by Wolstencroft & Simon (1975) to explain the variable circular polarization of the pre-main sequence star V1057 Cygni. The alignment is a compromise between grains spinning normal to a magnetic field of spiral configuration which is convected outwards from the star by a low level stellar wind, as in the case of the interplanetary magnetic field, and grains spinning in a plane containing the radial direction and the normal to the stellar disc so as to present a minimum cross section to the stellar wind. The scattering phase function for a long cylindrical grain has a maximum on the surface of a cone whose axis coincides with the long axis of the grain (see figure 3 of Bandermann & Kemp 1973). As the grain spins, the half-angle of the cone, β, which is the angle between the incident beam and the grain axis, changes, but the time-averaged phase function is symmetrical with respect to the plane defined by the spinning grain rather than with respect to the direction of the incident beam, and this could be the root cause of the asymmetry. A preliminary calculation for the above geometry, where the plane defined by the rotation of the grain intersects the plane of the disc at an angle α to the radial direction, indicates a brightness ratio on the two sides of the disc of about 8% for $\alpha = 45\%$ and grain length $L = \lambda/2\pi$ (Wolstencroft & Dasgupta 1988). The sign of the asymmetry depends on whether α is measured clockwise or anticlockwise relative to the radial direction. For the case of imperfect alignment of such particles, or particles which are less elongated, the asymmetry would clearly be less and detailed calculations are required to determine whether the observed asymmetry is compatible with this mechanism given plausible grain shapes and alignment.

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

The surface brightness along the major axis of the disc falls off rapidly with projected distance from the star and Smith & Terrile (1984) showed, with a simple model of isotropically scattering dust grains, that this is consistent with a density law $n(r) \propto r^{-3}$. This conclusion has been challenged by Diner & Appleby (1986), who developed a model which fitted both the coronagraphic and IRAS results for β Pic and also used a more realistic scattering function: however, because they used IRAS fluxes that are too low and used an infrared emissivity inappropriate for small particles (Backman *et al.* 1988*b*) it is unclear whether their conclusions are valid. Buitrago & Mediavilla (1986) have also questioned the density law of Smith & Terrile. Using an inversion technique commonly used in zodiacal-light studies they show that a radial density law decreasing faster than $r^{-1.5}$ is improbable and conclude that the most likely law is $n(r) \propto r^{-1}$.

The disc has been imaged with B, V and I filters (0.45, 0.56 and 0.90 μm) by Gradie *et al.* (1987) and with B, V, R and I_c filters by Paresce & Burrows (1987), the latter between 5″ and 15″ (80 and 240 AU). Gradie *et al.* find that between 100 and about 400 AU the colour is constant with $(V-I) = 0.8$ for the disc and 0.2 for the star, yielding an 'intrinsic' colour of $(V-I) = 0.6$ for the disc, i.e. the disc is redder than the star; they also note that the colour may be bluer with 100 AU. On the other hand Paresce & Burrows find the disc colour to be neutral and constant between 100 and 190 AU, with intrinsic disc colours $B-I = -0.06$, $V-I = 0.02$ and $R-I = +0.08$ with an uncertainty of about 0.2 magnitudes. Although these two estimates of the (V–I) disc colour differ, they both indicate that the scattered light is not blue and therefore that the particles are not predominantly Rayleigh scatterers for which $2\pi a \ll \lambda$: the implication for particle size cannot be stated precisely but it is likely that grains with $a \lesssim 0.1$ μm are rare in the disc. Smith & Terrile, Gradie *et al.* and Paresce & Burrows all agree that the disc has a width of 50 AU. If the disc is precisely edge-on then the observed width corresponds to a range of orbital inclinations of approximately ±2.5°. If the disc is slightly inclined to the line of sight this represents an upper limit to the range of orbital inclinations.

Gradie *et al.* compare the optical properties of the material in the disc of β Pic and of solid material in the Solar System making use of both the V–I colour and the ratio of scattered to thermally emitted radiation. This latter quantity, which they term the model dependent reflectivity or 'albedo', is the ratio of $\int \nu F_\nu \, d\nu$ integrated (*a*) from 0.5 μm to 0.9 μm using the V and I images and (*b*) from 40 to 60 μm using the IRAS data. They quote the following values:

	β Pic	asteroids	cometary dust
intrinsic V–I	0.6[a]	0.3	0.2–0.5
'albedo'	0.2–0.3	0.04–0.50	0.05–0.20

[a] 0.0 ± 0.2 according to Paresce & Burrows (1987).

Their comparison with asteroids may not be fully justified unless the majority of the dust grains are very large ($2\pi a \gg \lambda$) which is unlikely to be so (see later in this section). Given the uncertainty of the intrinsic V–I colour of the disc, the comparison must be based on the 'albedo', and Gradie *et al.* conclude that the range in albedo of cometary dust is compatible with that of the disc of β Pic.

Infrared observations

The infrared excess deduced from pointed IRAS observations have been reported by Gillett (1986) and are shown for α Lyr, α PsA, β Pic and ε Eri in figure 1. The region containing the

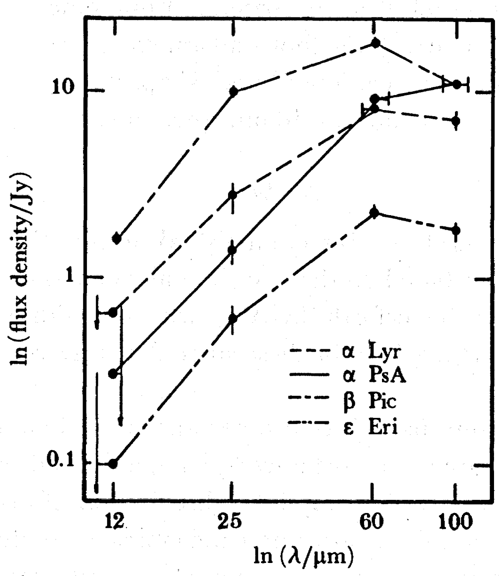

FIGURE 1. Infrared excess for the four prototype stars taken from Gillett (1986).

grains is likely to be quite extended, and thus have a wide range of temperatures, leading to a spectrum broader than that of a blackbody. Nevertheless, an approximate measure of the typical grain temperature may be deduced from the ratio of excess flux at 60 and 100 μm (T_{34}) and at 25 and 60 μm (T_{23}). These characteristic temperatures, derived by assuming that the grains radiate like black bodies, are in quite good agreement except for ε Eri.

star	β Pic	α Lyr	α PsA	ε Eri	reference
T_{23}/K	106	84	75	106	Aumann (1985)
T_{34}/K	110	85	60	45	D. E. Backman (personal communication)

Information on the angular extent of the prototype stars is available and is a vital constraint on the models. Scans of all four stars were made by the IRAS detector array, with the scans by the 60 μm detectors (285 × 91″) having the greatest signal:noise ratio. For β Pic, α Lyr and α PsA the scans across the star were made parallel to the short axis of the detector (in-scan) at one eighth of the normal scan rate and sampled every 3.6″. Limited information on the extent of the cloud perpendicular to the scan direction (cross-scan) comes from crossings by partially overlapping detectors in the array. Scans across a point source calibrator (α Boo) provided profiles (in-scan) with a full width at *quarter* maximum of 105″. ε Eri was observed only at one half of the normal scan rate. The angular extent at 60 μm (full width *half* maximum) after deconvolution is given by Gillet (1986).

star	β Pic	α Lyr	α PsA	ε Eri
position angle of scan (W of N)	0	5	29	20
FWHM (in-scan)/(arc s)	< 14	25±3	36±3	< 17
FWHM (cross-scan)	22±6	29±5	< 13	< 11

Because β Pic was only marginally resolved, perhaps because of the low signal:noise ratio of the IRAS slow scan data, and ε Eri was unresolved, other information is needed on the size of these two discs. Backman *et al.* (1988*b*) have carried out photometry of these stars at 3.4, 4.8, 10.1 and 20 μm in 4″ and 8″ apertures. β Pic shows an appreciable flux in excess of the predicted photospheric flux in the annulus between the 4″ and 8″ apertures both at 10 and 20 μm. ε Eri on the other hand shows no excess flux at 10 μm and only a 2σ excess at 20 μm (in both apertures).

Models

The models and results discussed in this section are taken from the study of all four stars by Backman *et al.* (1988*a*), which is based on the IRAS data alone, and from the study of β Pic and ε Eri by Backman *et al.* (1988*b*) (henceforth B.G.W.), which in addition to the IRAS data makes use of their 3–20 μm multi-aperture data for these stars. An early version of the basic model is described by Gillett (1986).

The model is an optically thin disc with an inner and outer boundary at distances r_1 and r_2 from the star. The disc is assumed to have a wedge shape defined by the rotation of a sector of small apex angle about the star. This implies a distribution of orbital inclinations for the grains that is independent of r. The size distribution and emissivity of the grains are also assumed to be independent of position in the disc and a power law is adopted to describe the number of grains per unit volume at distance r, namely $n \propto r^{-\gamma}$. The thermal emission from an individual grain depends on its equilibrium temperature which in turn depends both on its distance from the star and its emissivity, as well as on the stellar temperature. Grains are inefficient at either absorbing or emitting radiation at wavelengths λ much greater than the grain radius a, and to allow for this, the wavelength dependence of grain emissivity ε must be estimated. Although there is considerable variation in the emissivity curves for grains of likely composition and size (Draine & Lee 1984; Röser & Staude 1978), B.G.W. argue that a relation of the form $\epsilon = (1+\lambda/\lambda_0)^{-1}$ provides a fair analytical approximation: the parameter λ_0 is approximately equal to grain size a with the ratio $k = a/\lambda_0$ for likely materials lying in the range between about 0.1 and 2. The grains are assumed to be spinning rapidly and have a uniform surface temperature so that, given the assumption of an optically thin disc, the calculated flux is independent of the orientation of the disc to the line of sight.

Two types of model have been developed the first based on a single particle size and the second on a size distribution $n(a) \propto a^{-3.5}$. We discuss firstly the single particle size model. Briefly the model fitting is an iterative process that starts by assuming a value of λ_0 to deduce the dust temperature, T_d, as a function of r using the energy balance equation

$$\frac{\pi a^2}{4\pi r^2}\int_0^\infty B(\lambda, T_*)\,4\pi R_*^2\,\epsilon(\lambda)\,\mathrm{d}\lambda = 4\pi a^2\int_0^\infty B(\lambda, T_d)\,\epsilon(\lambda)\,\mathrm{d}\lambda,$$

where $B(\lambda, T)$ is the Planck function, T_d is the grain temperature and (T_*, R_*) are the stellar surface temperature and radius. The trial value of λ_0, together with assumed values of γ, r_1 and r_2, are used to calculate the integrated infrared excess from the dust disc, say at 60 μm, namely

$$F_{60} \propto \frac{1}{4\pi d^2}\int_{r_1}^{r_2}\int_{\lambda_1}^{\lambda_2} A_0\, r^{-\gamma} B(T_d, r)\,\epsilon(\lambda)\,4\pi r^2 \sin\theta_0\,\mathrm{d}r\,\mathrm{d}\lambda,$$

where $A_0 = n_0 \pi a^2$ is the total cross section of all grains at a reference distance from the star, d is the Earth–star distance and $\mathrm{d}V = 4\pi r^2 \sin\theta_0\,\mathrm{d}r$ is the volume element of the wedge-shaped disc defined by an angle $2\theta_0$ at the star. The IRAS slow scan profiles, or multi-aperture data in the case of β Pic and ε Eri, provide an additional and essential constraint on λ_0, r_1 and r_2 so that by iterative adjustment of parameters a good fit to the observations is obtained.

The model solutions of B.G.W. and Backman *et al.* (1988*a*) are given in table 1.

Table 1. Disc parameters for prototype stars ($n(r) \propto r^{-1}$)

star	λ_0/μm	r_1/AU	T_1/K	r_2/AU	T_2/K	L_d/L_*	A cm²
β Pic	0.8	17	210	790	45	3×10^{-3}	5×10^{30}
α PsA	80	26	115	450	32	9×10^{-5}	3×10^{28}
α Lyr	1100	22	165	245	50	3×10^{-5}	4×10^{27}
ε Eri	> 10	< 0.8	> 290	290	25	2×10^{-4}	—

The power-law exponent γ is not well constrained by the observations so that the choice of $\gamma = 1$ for these model solutions requires justification. This choice is based primarily on the following: (*a*) a dust cloud in equilibrium under the Poynting–Robertson effect, which is being continually replenished from a single source, has $\gamma = 1$ (Leinert *et al.* 1983); and (*b*) values of γ for the interplanetary dust cloud, obtained from model fits to the zodiacal light at optical and infrared wavelengths, lie in the range $\gamma = 1.0$ to 1.3. However, note that if the dust source providing replenishment exists in a very extended region then $\gamma > 1$ (Leinert 1985). If $\gamma = 2$ was adopted instead of $\gamma = 1$ the value of r_1 would increase by about 40%. The radius of the outer boundary is much more susceptible to the exact value of γ and is essentially undetermined for $\gamma > 2$. In the case of β Pic Buitrago & Mediavilla (1986) argue that $\gamma = 1$ is the most likely law and that the value of $\gamma = 3$ deduced by Smith & Terrile (1984) yields an implausible form for the scattering phase function for the dust.

Given the above caveats it is clear that the dust discs around β Pic, α PsA and α Lyr contain dust-free zones, which for $\gamma = 1$, lie at $r_1 = 17$ AU, 26 AU and 22 AU, respectively. The observations are not sensitive enough to exclude the presence of *any* dust in these zones but for example in the case of α Lyr the average number density of dust within 22 AU can be no more than 20% of the density at $r_1 = 22$ AU. For ε Eri the most probable value of r_1 is 0.8 AU although a model with r_1 as small as 0.05 AU is possible without violating the upper limit on the 12 μm flux observed by B.G.W.: at this distance the grain temperature (1000 K) is close to that at which refractory grains sublime.

The average grain sizes, based on $a = k\lambda_0$, where $0.1 \lesssim k \lesssim 2$, vary over three orders of magnitude for these four stars. If Poynting–Robertson drag were the dominant force affecting the orbits of dust grains in a given dust cloud, and if there was no replenishment of the dust cloud, unlike the case of the Solar System, then we would expect (i) the mean grain size to increase with time; and (ii) the total cross section of all grains in the cloud (A), the outer boundary radius and the ratio of dust cloud to stellar luminosity (L_d/L_*) to decrease with time. Although it is very unlikely that the original dust clouds around β Pic, α PsA and α Lyr were identical in all respects, it is remarkable that the values of λ_0, A, r_2 and L_d/L_* are consistent with an age sequence, in which α Lyr is the oldest and β Pic the youngest star. If true this would imply that the disc around these three A stars were initially 'similar', but clearly conclusions based on three examples cannot be taken too seriously.

B.G.W. and Backman *et al.* (1988*a*) have also developed models in which a power-law size distribution $n(a) \propto a^{-\delta}$ is used. They adopt $\delta = 3.5$, which describes the distribution of asteroid sizes (Dohnanyi 1969). Because Mathis *et al.* (1977) also advocate this size distribution for interstellar grains it does represent a reasonable first choice for a wide size range. For these models the minimum grain size, a_{min}, is used as a fitting parameter for size rather than λ_0 yielding values of a_{min} of 3.5 μm, 40 μm and more than about 0.5 μm for α PsA, α Lyr and ε Eri respectively. For β Pic no satisfactory fit is possible with this size distribution: a steeper size distribution is needed.

Earlier in this section we described the criteria used by Aumann (1985) to select the 12 stars within 25 pc most likely to show Vega-like discs. Walker & Wolstencroft (1988) (W.W.) have also done a search of the IRAS point source catalogue that uses similar criteria but is not limited to stars within 25 pc: their principal criteria are that (*a*) the 60–100 μm flux density ratio (in janskies†) is in the range defined by the prototype namely 0.8–2.0, and (*b*) there is evidence that the objects are extended based on the number of detectors that saw the source as it crossed the focal plane of IRAS. They find 16 stars with distances ranging from 30 to over 300 pc. The colour–colour plots (figures 2 and 3) illustrate that the [25]–[60] colours are similar for both samples, whereas the W.W. objects plus β Pic are well separated in [12]–[25] from the objects in Aumann's list. Selection effects are clearly important in favouring the detection of discs that

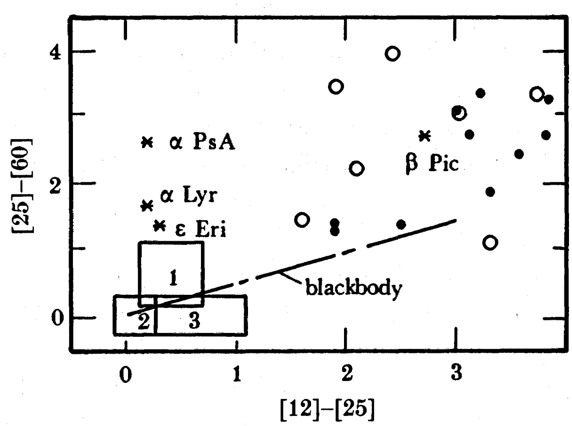

FIGURE 2. Infrared colours for the four prototypes and the sixteen W.W. objects. The boxes indicate the position of carbon stars (1), bright stars (2) and M stars (3). The open circles are objects which are either double or show emission lines in their optical spectra.

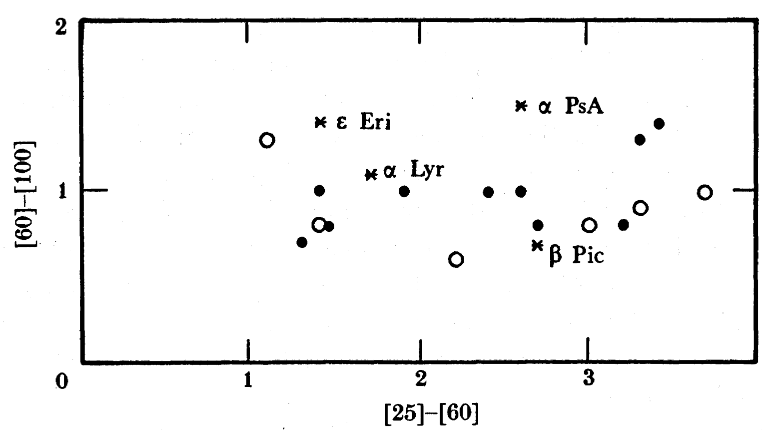

FIGURE 3. Infrared colours involving 25, 60 and 100 μm.

† 1 Jy = 10^{-26} Wm^{-2} Hz^{-1}.

are hot, large and massive in a sample without an upper limit of distance: nevertheless, 4 of the 16 stars in the W.W. list are within 80 pc and simple calculations by W.W. of the effective radius of these discs using the method described by Aumann (1985) indicate that one of these stars, 49 Ceti, may have a large disc. This star is also in the list of 12 stars published by Sadakane & Nishida (1986) although they used slightly different criteria from both W.W. and Aumann. The range of disc temperature and spectral type of the central stars for the 16 W.W. objects and 12 Aumann objects are shown in figure 4.

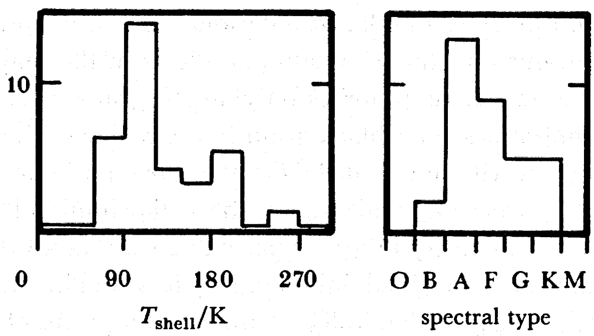

FIGURE 4. Histograms of mean dust cloud temperature (based on the 25–60 μm flux density ratio) and spectral class for the 12 objects listed by Aumann (1985) and the 16 W.W. objects.

4. DISCUSSION

(a) *Disc masses*

Only approximate estimates of the disc masses may be determined from the above model solutions. By integrating over the (assumed) grain size distribution ($\delta = 3.5$) we find that the mass in unit volume of the dust cloud at the reference distance is proportional to $\rho n_0(\sqrt{a_{max}} - \sqrt{a_{min}})$, where ρ is the mass density of the grain material and n_0 is the number density of grains at this distance. The uncertainty in the total disc mass is determined principally by the unknown value of a_{max}. For β Pic, for which no satisfactory model can be found for $\delta = 3.5$, a steeper size distribution ($\delta = 4.0$) has been used to calculate M_d. Using $\rho = 2$ gm cm^{-3}, two extreme values of a_{max}, and the values of a_{min} listed below, D. E. Backman (personal communication) has determined the following disc masses in units of $M_e = 1$ Earth mass.

TABLE 2. DISC MASSES

star	a_{min}	a_{max}	a_{lim}	M_d/M_e	δ
β Pic	1.6 μm	1 mm	2.3 μm	1	4.0
		100 km		5	
α PsA	3.5 μm	1 mm	4 μm	3×10^{-2}	3.5
		100 km		300	
α Lyr	40 μm	1 mm	14 μm	1×10^{-2}	3.5
		100 km		100	
ε Eri	0.5 μm	1 mm	0.24 μm	2×10^{-3}	3.5
		100 km		20	

The upper bound on a_{max} of 100 km, close to the maximum size for asteroids, yields disc masses between 5 and 300 M_e, much larger than the mass of Solar System bodies in this size range (*ca.* 10^{-3} M_e) and approaching the mass of the planets (450 M_e). If the gas:dust ratio was 100:1 by mass in the pre-main sequence disc and if little dust has been lost since that epoch, then the total disc mass at the pre-main sequence phase would have been in the range $0.006\ M_\odot$ to $0.09\ M_\odot$ for 3 of the 4 stars (excluding β Pic for which M_c is less certain): given all the uncertainties these values are not inconsistent with the estimate by Beckwith & Sargent (1987) of the total mass of the disc around HL Tau.

(b) *Grain removal and replenishment*

In the Solar System, the orbital evolution and lifetimes of particles is governed principally by the ratio, β, of radiation pressure to solar gravitational forces, Poynting–Robertson (P–R) drag, collisions between grains and the gravitational effects of the major planets. For small grains, solar-wind pressure and Lorentz forces on charged grains may also play a role (see Mukai 1985). For the simplest case of a black grain in a circular orbit, perhaps just released from a parent body, the grain will be expelled from the star on a hyperbolic orbit if $\beta > 0.5$, i.e. if $a < a_{\text{lim}} = 1.2 L_* / \rho M_*$ where L_* and M_* are the stellar luminosity and mass measured in solar units and ρ is the mass density in grams per cubic centimetre (Gillett 1986). For the three prototype stars with well determined values of $a_{\min}$ it is comforting to note (see table 2) that for $\rho = 2$ gm cm^{-3} $a_{\text{lim}} \lesssim a_{\min}$ (Gillett 1985). However, for grains of realistic composition, β reaches a maximum value above 0.5 and then falls below 0.5 to small values as the grain size is decreased (Burns *et al.* 1979): thus in principle very small grains ($a \lesssim 0.03$ μm) should survive. Because (P–R) drag is proportional to β these very small grains will not be removed rapidly from the disc. In the Solar System many of these very small grains are very probably removed by the solar-wind counterpart of the P–R drag because the ratio of solar wind pressure force to gravitational force continues to be proportional to $1/a$ for the smallest grains (Burns *et al.* 1979): for an obsidian grain with $a = 0.01$ μm which is initially at 1 AU, Mukai & Yamamoto (1982) show, that the P–R lifetime is less than 100 years allowing for the solar-wind component and greater than 10^5 years with only the solar-radiation term. This mechanism is effective also for ice grains, but not for metallic grains such as magnetite which may therefore survive. If this is the correct explanation for the absence of very small grains around the four prototype stars we must conclude that these stars possess a low-mass stellar wind.

For all but the smallest grains the orbital evolution is governed by the radiative component of P–R drag which removes both energy and angular momentum from the orbit of the grain, thus leading to a steady decrease in both the semimajor axis and eccentricity. To estimate approximate timescales for the circumstellar discs we assume circular orbits at all times. The time in years for the orbital radius of a grain to change from r_i to r_f (in astronomical units) is

$$T_{\text{PR}} = \frac{700 a \rho (r_i^2 - r_f^2)}{\eta (L_*/L_\odot)},$$

where the grain radius a is in micrometres, the density ρ is in grams per cubic centimetre and η is the fraction of the photon momentum transmitted to the grain (Burns *et al.* 1979). In table 3 we give the timescale for the smallest grains to spiral into the star from both the inner and outer boundaries of the dust cloud ($\eta = 1$ was assumed). Other relevant timescales shown in this table are the estimated age and main sequence lifetime, T_{ms}, of the star (Gillett 1986) and the orbital period, P, for a grain at the inner boundary. It is important to recognize that the remarks in this section about radiation pressure and P–R drag depend on the assumption that the grains are spherical. If allowance is made for the effect of radiation on non-spherical grains,

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

TABLE 3. TIMESCALES IN THE DISC

star	spectral type	$M_*/M_\odot$	$L_*/L_\odot$	$T(r_1)$ years	$T(r_2)$ years	$P(r_1)$ years	T_{ms} years	age years
β Pic	A5V	1.7	6.5	1.0×10^5	2.2×10^8	54	2×10^9	—
α PsA	A3V	1.9	13	2.5×10^5	7.5×10^7	96	1×10^9	—
α Lyr	A0V	2.5	60	4.5×10^5	5.6×10^7	65	5×10^8	$\sim 5\times10^8$
ε Eri	K2V	0.9	0.37	$<2.2\times10^2$	2.9×10^7	0.8	2×10^{10}	$\sim 1\times10^9$

the value of β as a function of grain volume can be appreciably different from the spherical case: Voshchinnikov & Ilin (1983) show for example that small ($a \lesssim 0.1$ μm) cylindrical grains will have much larger values of β than spherical grains.

How do these timescales bear on the question of the replenishment of these discs? If all discs began with the same outer radius $r_1 = 1000f$ AU (f is a constant of order unity), the same wide range of sizes and no replenishment took place, the time for P–R drag to sweep out all grains smaller than $a_{\min}$ would be 9.3×10^8 (α Lyr), 3.8×10^8 (α PsA), 3.5×10^8 (β Pic) and 1.9×10^9 (ε Eri) in units of f^2 years. For $f^2 \approx \frac{1}{2}$ these times are consistent with the estimated ages of α Lyr and ε Eri (table 3) and plausible for β Pic and α PsA. In other words, there is no *prima facie* case for a balance between sources and sinks of dust, because discs initially of radius *ca.* 1000 AU or greater (compatible with that observed for HLTau) can survive the P–R depletion. If replenishment does occur, then the absence of grains with $a \lesssim 40$ μm in Vega's disc would imply that small grains continually injected into the disc must be removed by some other process, or that no small grains are fed into the disc. If collisions occur, either erosive or catastrophic (Dohnanyi 1978) then they would produce more small grains and although some may be small enough to be removed rapidly by radiation pressure, solar wind pressure or P–R drag, collisional debris may in general make the problem of the observed absence of small grains more serious. The interrelation of the various dynamical and collisional processes, dust sources and sinks, and the size and spatial distribution of the grain population may be quite complex, as can be judged from the study of the replenishment of the interplanetary dust cloud by Leinert *et al.* (1983). In that case they show that the consequence of replenishing the cloud under the combined influence of P–R drag and collisional processes is to produce a density law close to $n(r) \propto r^{-1}$. The models of optical surface brightness of the β Pic disc by Buitrago & Mediavilla (1986) have shown that $n(r) \propto r^{-1}$ is more realistic than the law $n(r) \propto r^{-3}$ proposed by Smith & Terrile (1984): the study by Leinert *et al.* implies that both replenishment and P–R drag are necessary to give the observed value of the power law exponent and to some extent these arguments support the idea that both P–R drag and replenishment are occurring in the β Pic disc.

The presence of a dust-free zone at $r \lesssim 25$ AU surrounding α Lyr, α PsA and β Pic is a striking result. P–R drag should supply the inner zone quickly especially with small grains: why does this not happen? A clue to this comes from the study by Leinert *et al.* (1983) who show how the steady-state density distribution $n(r)$ depends on the ratio of P–R and collisional lifetimes, τ_{PR}/τ_c, and on the orbital distribution, $f(a, e)$, of dust sources: they find that if collisions dominate ($\tau_{PR}/\tau_c \gtrsim 3$) and the source distribution ceases at some distance R_0 from the star then $n(r)$ reaches a maximum at R_0 and decreases monotonically at $r < R_0$ (see their figure 12). This is because grains are destroyed in catastrophic collisions before they reach the star under the influence of P–R drag. Whether these conditions could be realized in the case of any

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

of the stellar discs needs to be investigated. It requires *inter alia* that the orbits of potential dust sources, such as comets, asteroids or first-order collisional debris from either, would not exist within r_1. This could be an accident of formation in the case of asteroids whose relatively circular orbits (drawing a Solar System analogy) would be reasonably immune to perturbations by planets that were trying to move them into orbits closer to the star. For comets formed far out in the protostellar cloud a deficit of comets within *ca.* 25 AU would only make sense if major planets did not exist to gravitationally capture a significant fraction into 'short period' comets: it is unclear whether the formation of comets but not planets is a plausible scenario for a stellar disc. Another possible explanation of the dust-free zone (for the case of β Pic) has been made by B.G.W. who suggest that an object, possibly a planet or protoplanet, orbits the star just inside the inner boundary: this object would act to 'shepherd' or sweep up particles as they try to spiral in past its orbit. They attribute the absence of a dust-free zone around ε Eri to the absence of a shepherding object: however, the tentative detection of an unseen companion around ε Eri (with $P \gtrsim 6$ years and $r > 3.2$ AU) by Campbell *et al.* (1987), if confirmed, would argue against this explanation. A third possible explanation for the dust-free zone is that the inner disc boundary is a region at which water-ice grains sublime. Isobe (1970) has computed the time, t_s, for ice grains to sublime at temperature T_d. The vapour pressure for an ice grain depends critically on T_d in the range 90 to 110 K and the corresponding lifetime t_s for grains of radius 1 (10) μm are 80 (800) Ma at 90 K and 3 (30) ka at 110 K. At the inner boundaries of the three A stars with dust-free zones, the value of t_s for a grain of radius a_{min} is measured in seconds for β Pic (210 K), weeks for α Lyr (165 K) and is about 1 ka for α PsA (115 K). This suggests that icy grains with $a \lesssim 10\ a_{min}$ do not exist near the inner boundary of the β Pic and α Lyr discs but that they could exist in the α PsA disc. Thus only for α PsA is ice sublimation a possible explanation for the dust-free zone.

If replenishment of any or all of these discs is taking place, grains ejected from comets are the most obvious source. Harper *et al.* (1984), citing Sekanina (1975), suggest that the average distance at which comets start ejecting dust on their inward journey or stop ejecting on their outward journey around the Sun could be as high as 10 AU, which is thus equivalent in terms of temperature to that found at a distance from Vega of $10(L_*/L_\odot)^{\frac{1}{2}}$ AU ≈ 80 AU. Hence the inner parts of the Vega disc could be replenished by cometary grains. For ε Eri, β Pic and α PsA the corresponding zone lies at 6 AU, 25 AU and 36 AU from the star, so that the contribution of dust ejected from comets around these stars would be much less significant than for Vega. Weissman (1984) has suggested that the outer parts of the Vega disc may have been replenished as the result of collisions between cometesimals, the parent bodies of comets, and it is possible that this process could replenish the bulk of the β Pic and α PsA discs. Weissman also comments on the possibility that parts of these discs may be analogous to the inner Oort cloud, which has been postulated by Cameron (1978) to have formed in a planetary accretion disc beyond the orbit of Neptune out to a distance of several hundred astronomical units. These suggestions need to be developed in considerable detail before definitive conclusions can be drawn about the replenishment of these discs.

We are greatly indebted to Dr D. Backman for much helpful discussion and for allowing us to quote from his papers before publication. We also thank Professor B. Zuckerman and Dr D. Whitmire for useful comments. H.J.W. thanks the NASA Ames Research Center who provide support for IRAS studies through the SETI Institute under cooperative agreement NCC2-407.

References

Aumann, H. H., Gillett, F. C., Beichman, C. A., de Jong, T., Houck, J. R., Low, F. J., Neugebauer, G., Walker, R. G. & Wesselius, P. R. 1984 *Astrophys. J.* **278**, L23–L27.
Aumann, H. H. 1985 *Publ. astr. Soc. Pacif.* **97**, 885–891.
Backman, D. E., Gillett, F. C., Aumann, H. H., Neugebauer, G., Low, F. J. & Waters, R. 1988*a* (In preparation.)
Backman, D. E., Gillett, F. C. & Witteborn, F. C. 1988*b* *Astrophys. J.* (Submitted.).
Bandermann, L. W. & Kemp, J. C. 1973 *Mon. Not. R. astr. Soc.* **162**, 367–377.
Beckwith, S. & Sargent, A. 1987 *Circumstellar matter* (ed. I. Appenzeller & C. Jordan), pp. 81–83. Dordrecht: Reidel.
Black, D. C. & Matthews, M. S. (eds) 1985 Protostars and planets II. Tucson: University of Arizona Press.
Buitrago, J. & Mediavilla, E. 1986 *Astron. Astrophys.* **162**, 95–98.
Burns, J. A., Lamy, P. L. & Soter, S. 1979 *Icarus* **40**, 1–48.
Cameron, A. G. W. 1978 *The origin of the Solar System* (ed. S. F. Dermott), pp. 49–75. New York: Wiley.
Campbell, B., Walker, G. A. H. & Yang, S. 1987 In *Proc. IAU Colloquium 99 on Bioastronomy: The Next Steps, June 1987, Lake Balaton, Hungary* (ed. J. Tarter). (See also *Sky Telescope*, p. 125 (August 1987).)
Cassen, P. M. & Summers, A. L. 1983 *Icarus* **53**, 26–40.
Cassen, P., Shu, F. H. & Terebey, S. 1985 In *Protostars and planets II* (ed. D. C. Black & M. S. Matthews), pp. 448–483. Tucson: University of Arizona Press.
Cohen, M. 1983 *Astrophys. J.* **270**, L69–L71.
Diner, D. J. & Appleby, J. F. 1986 *Nature, Lond.* **322**, 436–438.
Dohnanyi, J. S. 1969 *J. geophys. Res.* **74**, 2531–2554.
Dohnanyi, J. S. 1978 In *Cosmic dust* (ed. J. A. M. McDonnell), pp. 527–605. New York: Wiley.
Draine, B. T. & Lee, H. M. 1984 *Astrophys. J.* **285**, 89–108.
Gillett, F. C. 1986 In *Light on dark matter* (ed. F. Israel), pp. 61–69. Dordrecht: Reidel.
Gliese, W. 1969 *Catalogue of nearby stars*, no. 22. Heidelberg: Publ. Astr. Rechen-Institute.
Good, J. C. 1987 IRAS preprint, Infrared Process and Analysis Department, Pasadena.
Gradie, J., Hayashi, J., Zuckerman, B., Epps, H. & Howell, R. 1987 *Proc. 18th Lunar and Planetary Science Conference (Part I)*, pp. 351–352 (abstract).
Harper, D. A., Loewenstein, R. F. & Davidson, J. A. 1984 *Astrophys. J.* **285**, 808–812.
Hildebrand, R. H. 1983 *Q. J. R. astr. Soc.* **24**, 267–282.
Hollis, J. M., Chin, G. & Brown, R. L. 1985 *Astrophys. J.* **294**, 646–648.
Isobe, S. 1970 *Publs astr. Soc. Japan* **22**, 429–445.
Leinert, C., Röser, S. & Buitrago, J. 1983 *Astron. Astrophys.* **118**, 345–357.
Leinert, C. 1985 In *Proc IAU Colloq. 85 on Properties and Interactions of Interplanetary Dust* (ed. R. H. Giese & P. Lamy), pp. 369–375. Dordrecht: Reidel.
Mathis, J. S., Rumpl, W. & Nordsieck, K. H. 1977 *Astrophys. J.* **217**, 425–433.
Mukai, T. 1985 *Astron. Astrophys.* **153**, 213–217.
Mukai, T. & Yamamoto, T. 1982 *Astron. Astrophys.* **107**, 97–100.
Mundt, R. S. & Fried, J. W. 1983 *Astrophys. J.* **274**, L83–L86.
Myers, P. C. & Benson, P. J. 1983 *Astrophys. J.* **266**, 309–320.
Paresce, F. & Burrows, C. 1987 *Astrophys. J.* **319**, L23–L25.
Parker, E. N. 1979 *Cosmical magnetic fields*. Oxford University Press.
Röser, S. & Staude, H. J. 1978 *Astron. Astrophys.* **67**, 381–394.
Rydgren, A. E. & Cohen, M. 1985 In *Protostars and planets II* (ed. D. C. Black & M. S. Matthews), pp. 371–385. Tucson: University of Arizona Press.
Sadakane, K. & Nishida, M. 1986 *Publ. astr. Soc. Pacific* **98**, 685–689.
Sekanina, Z. 1975 *Icarus* **25**, 218–238.
Shu, F. & Adams, F. C. 1987 In *Circumstellar matter* (ed. I. Appenzeller & C. Jordan), pp. 7–22. Dordrecht: Reidel.
Smith, B. A. & Terrile, R. J. 1984 *Science, Wash.* **226**, 1421–1424.
Voshchinnikov, N. V. & Ilin, V. B. 1983 *Soviet Astron.* **27**, 650–654.
Walker, H. J. & Wolstencroft, R. D. 1988 *Publ. astr. Soc. Pacific.* (Submitted.)
Walker, C. K., Lada, C. J., Young, E. T., Maloney, P. R. & Wilking, B. A. 1986 *Astrophys. J.* **309**, L47–L51.
Ward-Thompson, D., Warren-Smith, R. F., Scarrott, S. M. & Wolstencroft, R. D. 1985 *Mon. Not. R. astr. Soc.* **215**, 537–544.
Weissman, P. R. 1984 *Science, Wash.* **224**, 987–989.
Whitmire, D. P., Tomley, L. & Matese, J. J. 1987 *Bull Am. Astr. Soc.* **19**, 830.
Wolstencroft, R. D. & Simon, T. 1975 *Astrophys. J.* **199**, L169–L171.
Wolstencroft, R. D. & Dasgupta, A. 1988 (In preparation.)
Woolley, R., Epps, E. A., Penston, M. J. & Pocock, S. B. 1970 *Royal Obs. Annal.* no. 5.